Theory of Impossibility

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Abstract

We propose a formal theory in which a single ontological prohibition — the impossibility of effective collapse of distinction — generates a unified structure of observation, resource, obstruction, and irreducible residue. The theory is deliberately split into a core layer and a model layer: the core fixes distinction, observational equivalence, and resource classes, while the model layer introduces scalar cost, obstruction to gluing, and the central law. Standard no-go results such as no-free-erasure, no-universal-reduction, no-free-simulation, and no-cloning follow as consequences. Information, computation, geometry, time, and layered scientific descriptions are interpreted as model-level realizations of the same structural constraint. The theory is falsifiable: a single realized instance of effective collapse without residue and without cost would refute it.

1 Introduction

Most foundational theories begin with positive ontological primitives: space, time, fields, states, observers, or information. This is methodologically effective, but it leaves open a more general question: what is primary — the inventory of entities, or the boundary of their admissible reduction?

The present theory proceeds in the opposite direction. It takes as its starting point not a thing, but a prohibition: the impossibility of completely and effectively eliminating distinction. This prohibition is not a logical tautology and does not reduce to the law of non-contradiction. It is an ontological constraint on admissible structures and admissible transformations of structures.

The aim is to derive from a single fundamental prohibition a cascade of consequences concerning observation, symmetry, reversibility, cost, information, computation, and, at the model level, geometric and physical interpretation. Numerical quantities, metrics, and scientific domains are introduced only as model layers, not as part of the ontological core.

The structure of the theory is as follows:

1. first, a language of distinction and observation is introduced;
2. then a resource category is constructed as a quotient by observational congruence;
3. then a model-level scalarization of cost is defined;
4. then the single axiom is stated;
5. then the central law is derived;
6. then standard no-go statements and model interpretations are obtained.

2 Core: Observation and Distinction

2.1 Category of Processes

Let E be a non-empty category. Objects of E represent configurations or states, and morphisms represent admissible transitions between states.

We do not assume additional structure at the core level. Composition and identity are sufficient for the formal role played by the category.

2.2 Observational Profile

Let $A \subseteq \mathrm{Ob}(E)$ be a small generating family of "stages."

Definition 2.1 (Local observation)**.** *For $U \in A$ and $X \in \mathrm{Ob}(E)$, define*

$$\mathrm{Obs}_U(X) := \mathrm{Hom}_E(U, X).$$

Definition 2.2 (Observational profile)**.** *The observational profile of an object X is the family*

$$\mathrm{Obs}(X) := \big(\mathrm{Obs}_U(X)\big)_{U \in A}.$$

Thus, an object is accessed not directly, but through all admissible local probes.

Lemma 2.3. *The assignment $X \mapsto \mathrm{Obs}(X)$ defines a functor*

$$\mathrm{Obs} : E \to \prod_{U \in A} \mathbf{Set}.$$

Proof. Let $f : X \to Y$. For each $U \in A$, define

$$\mathrm{Obs}_U(f) : \mathrm{Hom}_E(U, X) \to \mathrm{Hom}_E(U, Y), \quad s \mapsto f \circ s.$$

Then

$$\mathrm{Obs}_U(\mathrm{id}_X)(s) = s, \qquad \mathrm{Obs}_U(g \circ f)(s) = g \circ f \circ s = \mathrm{Obs}_U(g)\big(\mathrm{Obs}_U(f)(s)\big).$$

Hence Obs is functorial. □

2.3 Observational Equivalence of Objects

Definition 2.4. *Objects X, Y are observationally equivalent if*

$$X \sim_{\mathrm{obj}} Y \iff \mathrm{Obs}(X) \cong \mathrm{Obs}(Y),$$

where the isomorphism is componentwise over all $U \in A$.

Lemma 2.5. $\sim_{\mathrm{obj}}$ *is an equivalence relation.*

Proof. This follows from the reflexivity, symmetry, and transitivity of isomorphism in **Set**. □

2.4 Observational Congruence of Morphisms

Definition 2.6. *For morphisms $f, g : X \to Y$, define*

$$f \sim g \iff \forall U \in A,\ \forall s : U \to X, \quad f \circ s = g \circ s.$$

Two morphisms are equivalent if no admissible local test distinguishes them.

Lemma 2.7. *$\sim$ is a congruence relation on morphisms.*

Proof. Reflexivity, symmetry, and transitivity follow from equality.

To check compatibility with composition, let $f_1 \sim f_2 : X \to Y$ and $g_1 \sim g_2 : Y \to Z$. For any $s : U \to X$,

$$f_1 \circ s = f_2 \circ s.$$

Therefore,

$$g_1 \circ f_1 \circ s = g_1 \circ f_2 \circ s.$$

Since $g_1 \sim g_2$, we have $g_1 \circ t = g_2 \circ t$ for every $t : U \to Y$, and in particular for $t = f_2 \circ s$. Hence

$$g_1 \circ f_1 \circ s = g_2 \circ f_2 \circ s.$$

Thus $g_1 \circ f_1 \sim g_2 \circ f_2$. □

2.5 Resource Category

Definition 2.8. *The resource category is the quotient*

$$V := E/\sim.$$

Its objects are the objects of E, and its morphisms are equivalence classes of morphisms in E.

Definition 2.9. *For a morphism f, define its resource class*

$$\mathrm{Cost}(f) := [f] \in \mathrm{Mor}(V).$$

At this level, $\mathrm{Cost}(f)$ is not yet numerical. It is an observationally invariant transformation type.

3 Model Layer: Cost Structure

Numerical cost is introduced as a model-level scalarization of the resource category V.

Definition 3.1 (Scalarization family)**.** *Let M be a family of monoidal functionals*

$$\mu : V \to \mathbb{R}_{\geq 0}$$

such that:

1. *$\mu(I) = 0$,*
2. *$\mu(v \otimes w) = \mu(v) + \mu(w)$,*
3. *if $v \neq I$, then*

$$\inf_{\mu \in M} \mu(v) > 0.$$

Condition (3) is a separation condition.

Remark 3.2. *$\kappa^-(f)$ represents the minimal unavoidable cost. $\kappa^+(f)$ represents the realized or accumulated cost.*

Definition 3.3 (Lower and upper cost)**.** *Define*

$$\kappa^-(f) := \inf_{\mu \in M} \mu(\mathrm{Cost}(f)), \qquad \kappa^+(f) := \sup_{\mu \in M} \mu(\mathrm{Cost}(f)).$$

Lemma 3.4. *For every morphism f,*

$$\kappa^-(f) \geq 0, \qquad \kappa^+(f) \geq 0.$$

Proof. Each $\mu(\mathrm{Cost}(f))$ is nonnegative. Hence its infimum and supremum are nonnegative. □

Lemma 3.5. *If $\kappa^-(f) = 0$, then $\mathrm{Cost}(f) = I$.*

Proof. If $\mathrm{Cost}(f) \neq I$, then the separation condition implies a strictly positive lower bound on $\mu(\mathrm{Cost}(f))$, contradicting $\kappa^-(f) = 0$. □

Lemma 3.6. *If $\kappa^+(f) = 0$, then $\mathrm{Cost}(f) = I$.*

Proof. The supremum of a family of nonnegative numbers can be zero only if every element of the family is zero. By the separation condition, this happens only for the neutral class I. □

Definition 3.7 (Symmetry)**.** *A morphism f is a symmetry if*

$$\mathrm{Cost}(f) = I.$$

Lemma 3.8. *For every morphism f, the following are equivalent:*

1. *f is a symmetry;*
2. *$\kappa^-(f) = 0$;*
3. *$\kappa^+(f) = 0$.*

Proof. The implications (1) ⇒ (2) and (1) ⇒ (3) follow from $\mu(I) = 0$ for all $\mu \in M$. The implication (2) ⇒ (1) follows from the previous lemma. The implication (3) ⇒ (1) follows from the previous lemma. □

Corollary 3.9. *The class of symmetries is closed under composition.*

Proof. If f and g are symmetries, then $\mathrm{Cost}(f) = I$ and $\mathrm{Cost}(g) = I$. Therefore

$$\mathrm{Cost}(g \circ f) = \mathrm{Cost}(g) \otimes \mathrm{Cost}(f) = I \otimes I = I.$$

Hence $g \circ f$ is again a symmetry. □

4 Model Layer: Obstruction to Gluing

The core theory does not require a specific cohomology theory. However, to express the failure of local data to assemble into a global object, we fix a model layer in which such a failure is represented by an obstruction class.

4.1 Coverings and Matching Families

Let $X \in \mathrm{Ob}(E)$, and let

$$\mathcal{U} = \{u_i : U_i \to X\}_{i \in I}$$

be a chosen covering family in the model.

A family of local data $s = (s_i)_{i \in I}$, where

$$s_i \in \mathrm{Hom}_E(U_i, X),$$

is called compatible if it satisfies the usual matching condition on overlaps, relative to the chosen model of coverings.

4.2 The Gluing Map

In the model layer, for each covering $\mathcal{U}$ of X, we assume a canonical map

$$g_{X,\mathcal{U}} : \mathrm{Hom}_E(X, X) \longrightarrow \mathrm{Match}(X, \mathcal{U}),$$

where $\mathrm{Match}(X, \mathcal{U})$ denotes the set of matching families over $\mathcal{U}$.

The map $g_{X,\mathcal{U}}$ sends a global object to its induced local family. The precise construction of $\mathrm{Match}(X, \mathcal{U})$ is model-dependent. What matters is that it exists in the chosen model and is functorial in the expected sense.

4.3 Definition of the Obstruction Class

Definition 4.1 (Obstruction class)**.** *For a matching family $s \in \mathrm{Match}(X, \mathcal{U})$, define the obstruction class $\delta(s)$ as the class measuring whether s lies in the image of the gluing map $g_{X,\mathcal{U}}$.*

More precisely,

$$\delta(s) = 0 \iff s \in \mathrm{Im}(g_{X,\mathcal{U}}),$$

and

$$\delta(s) \neq 0 \iff s \notin \mathrm{Im}(g_{X,\mathcal{U}}).$$

Thus, $\delta(s)$ is not a primitive entity. It is the model-theoretic obstruction to lifting a compatible local family to a global one.

Remark 4.2. *In the chosen model, $\delta(s)$ plays the role of an obstruction class analogous to obstruction classes in sheaf theory, cohomological gluing problems, and related categorical lifting problems. The present theory does not require a specific cohomology theory; it requires only that the obstruction distinguish liftable from non-liftable compatible families.*

4.4 The Obstruction Object

Let

$$\mathcal{O}(X, \mathcal{U})$$

be a pointed obstruction object with distinguished element 0.

The obstruction class $\delta(s)$ takes values in $\mathcal{O}(X, \mathcal{U})$ and satisfies:

$$\delta(s) = 0 \iff s \text{ is globally liftable},$$

$$\delta(s) \neq 0 \iff s \text{ is not globally liftable}.$$

Lemma 4.3. *If the obstruction set is trivial, then every compatible local family globally glues.*

Proof. If there is no nonzero obstruction class, then every compatible local family lies in the image of the gluing map by definition of δ. Hence every compatible family is globally liftable. $\square$

Lemma 4.4. *If f is a symmetry, then*

$$\delta(f \circ s) = \delta(s).$$

Proof. A symmetry has zero cost and preserves the observational class. Since δ only records whether a compatible family lifts globally, and symmetries preserve the relevant observational and resource class, the obstruction is unchanged. $\square$

Lemma 4.5. *If $\kappa^{-}(f) = 0$, then f cannot remove a nonzero obstruction.*

Proof. If $\kappa^{-}(f) = 0$, then f is a symmetry by the cost lemma. By the previous lemma, symmetries preserve δ. Hence a nonzero obstruction cannot become zero under such an f. $\square$

5 Axiom of Effective Collapse

Definition 5.1 (Effective collapse)**.** *An effective collapse is a reduction process that sends every observationally meaningful structure to a single trivial observational class, while preserving the formal compositional apparatus and eliminating all residual distinction without cost.*

[Axiom of impossibility]

$$\neg 0_{\mathrm{eff}}(E).$$

This axiom states that effective collapse is impossible. Equivalently, no admissible reduction can eliminate all distinction without residual cost.

Lemma 5.2. *If $0_{\mathrm{eff}}(E)$ holds, then all observational profiles become indistinguishable.*

Proof. By definition, effective collapse maps every observationally meaningful structure to one trivial class. Hence all profiles coincide. $\square$

Lemma 5.3. *If for some family s and morphism f one has*

$$\Delta_H(f \circ s) = 0 \quad \textit{and} \quad \kappa^{-}(f) = 0,$$

then $0_{\mathrm{eff}}(E)$ holds.

Proof. The condition $\kappa^{-}(f) = 0$ implies that f is a symmetry. The condition $\Delta_H(f \circ s) = 0$ means that the obstruction vanishes after applying f, so the family globally lifts without residual obstruction. Thus f simultaneously acts without resource cost and removes the obstruction. This is exactly an effective collapse. $\square$

Theorem 5.4. *It is impossible to annihilate both obstruction and cost simultaneously.*

Proof. Assume that there exist f and s such that

$$\Delta_H(f \circ s) = 0 \quad \text{and} \quad \kappa^{-}(f) = 0.$$

Then, by the previous lemma, $0_{\mathrm{eff}}(E)$ holds. This contradicts the axiom $\neg 0_{\mathrm{eff}}(E)$. Therefore, the simultaneous vanishing of obstruction and cost is impossible. $\square$

Corollary 5.5. *Distinction cannot be removed for free.*

Proof. If distinction were removable for free, then there would exist a morphism of zero lower cost eliminating a nonzero obstruction. This is forbidden by the theorem above. $\square$

6 Central Law of Irreducible Residue

The axiom and the model-layer definitions now yield the central law.

Definition 6.1. *For a compatible family s, define the scalarized obstruction by*

$$\Delta_H(s) := \inf_{\mu \in M} \mu(\delta(s)).$$

Lemma 6.2.

$$\delta(s) = 0 \implies \Delta_H(s) = 0.$$

Proof. If $\delta(s) = 0$, then $\mu(\delta(s)) = \mu(0) = 0$ for every $\mu \in M$. Hence the infimum is zero. □

Lemma 6.3.

$$\Delta_H(s) = 0 \implies \delta(s) = 0.$$

Proof. If $\delta(s) \neq 0$, then separation of M implies a strictly positive lower bound on $\mu(\delta(s))$, contradicting $\Delta_H(s) = 0$. Therefore $\delta(s) = 0$. □

Theorem 6.4 (Central law)**.** *For every admissible morphism f and every compatible family s,*

$$\Delta_H(f \circ s) + \kappa^-(f) > 0.$$

Proof. Assume, for contradiction, that

$$\Delta_H(f \circ s) = 0 \quad \text{and} \quad \kappa^-(f) = 0.$$

By the preceding lemmas, this implies

$$\delta(f \circ s) = 0 \quad \text{and} \quad \mathrm{Cost}(f) = I.$$

Hence f is a symmetry and the family $f \circ s$ is globally liftable without cost.

At that point the model has eliminated both the obstruction and the resource price of the same nontrivial distinction. That is exactly what the axiom $\neg 0_{\mathrm{eff}}(E)$ forbids: a process that collapses observably meaningful structure to a trivial class while leaving no residue and no cost. Therefore the assumption is impossible, and

$$\Delta_H(f \circ s) + \kappa^-(f) > 0.$$

□

Corollary 6.5. *A nonzero obstruction cannot be eliminated at zero cost.*

Corollary 6.6. *Any process that removes distinction must pay a positive lower cost.*

7 Information and Computation

The central law states that distinction cannot be eliminated for free. We now interpret this structural constraint on two further levels: information and computation. These are not new ontological primitives. They are two model-level modes of handling distinction.

7.1 Information

Definition 7.1 (Information)**.** *Information is a stable distinction, that is, a nonzero obstruction class that remains invariant under admissible transformations of the observational profile.*

In other words, information is not a carrier, a message, or a code in the first instance. Information is distinction that cannot be removed without resource cost.

Lemma 7.2. *If f is a symmetry, then for every compatible family s,*

$$\delta(f \circ s) = \delta(s).$$

Proof. This is a direct consequence of the invariance of δ under admissible re-descriptions and the fact that symmetries preserve the observational class. □

Theorem 7.3. *Free erasure of information is impossible.*

Proof. Assume that there exists a morphism f with $\kappa^-(f) = 0$ such that

$$\delta(f \circ s) = 0$$

for some family s with $\delta(s) \neq 0$. Then f is a symmetry. By the previous lemma, symmetries preserve the obstruction class, so

$$\delta(f \circ s) = \delta(s) \neq 0,$$

contradiction. Hence no zero-cost morphism can erase information. □

Corollary 7.4. *Any operation that removes information has positive lower cost.*

7.2 Computation

Definition 7.5 (Computation)**.** *A computation is an admissible morphism $f : X \to Y$ that changes the observational profile:*

$$\mathrm{Obs}(X) \ncong \mathrm{Obs}(Y).$$

Definition 7.6 (Algorithm)**.** *An algorithm is a finite composition*

$$A = f_n \circ \cdots \circ f_1$$

of admissible morphisms such that at least one nontrivial step changes the observational profile.

Lemma 7.7. *If $\kappa^-(f) = 0$, then f does not change the observational profile.*

Proof. If $\kappa^-(f) = 0$, then f is a symmetry. Symmetries preserve the observational class. Therefore the profile is unchanged. □

Theorem 7.8. *Any nontrivial computation has strictly positive lower cost.*

Proof. Suppose that f changes the observational profile and that $\kappa^-(f) = 0$. By the lemma above, f would preserve the profile. This contradicts the assumption that f is computationally nontrivial. Therefore $\kappa^-(f) > 0$. □

Corollary 7.9. *Free computation that genuinely changes distinction is impossible.*

Theorem 7.10. *Information and computation are two modes of the same structural constraint.*

Proof. Information is the stable presence of a nonzero obstruction class. Computation is an admissible process that transforms observational profiles, hence acts on the same structural data. In both cases, the relevant objects are distinctions and their admissible transformations. Therefore information and computation are not different kinds of being; they are two model-level descriptions of the same underlying constraint. □

8 Geometry and Time

The theory now admits a geometric and temporal reading. This is a model interpretation, not a new ontological layer.

8.1 Geometry

Let a path from X to Y be a finite composition of admissible morphisms

$$X = X_0 \to X_1 \to \cdots \to X_n = Y.$$

For such a path define its length by

$$L = \sum_{i=1}^{n} \kappa^+(f_i).$$

Definition 8.1 (Distance). *Define the distance between X and Y by*

$$d(X,Y) := \inf L,$$

where the infimum is taken over all admissible paths from X to Y.

Here the upper cost κ^+ is used because path length is a realized accumulated cost, not a minimal unavoidable one.

Lemma 8.2. *For all X, Y,*

$$d(X,Y) \geq 0.$$

Proof. Every path length is a sum of nonnegative terms. Hence every path length is nonnegative, and so is the infimum. □

Lemma 8.3. *For every object X,*

$$d(X,X) = 0.$$

Proof. Consider the trivial path consisting only of the identity morphism. Its length is zero because $\kappa^+(\mathrm{id}_X) = 0$. Therefore the infimum over all paths from X to X is zero. □

Theorem 8.4. *Assuming attainment of the infimum,*

$$d(X,Y) = 0 \quad \Longleftrightarrow \quad X \sim_{\text{obj}} Y.$$

Proof. If $d(X,Y) = 0$, then there exists a sequence of admissible paths whose lengths approach zero. Since all summands are nonnegative, each nontrivial component must have zero upper cost. By the equivalence established earlier, such components are symmetries. Compositions of symmetries are symmetries, hence X and Y are observationally equivalent.

Conversely, if $X \sim_{\text{obj}} Y$, then in the model there is a zero-cost realization through a resource-trivial class, and hence $d(X,Y) = 0$. □

Corollary 8.5. *Geometry arises as a structure of cost on admissible paths.*

Corollary 8.6. *Space is not a primitive entity; it is a derived form of accessibility between distinctions.*

8.2 Time

Definition 8.7 (Time of a process). *Define the time of a morphism f by*

$$t(f) := \kappa^{+}(f).$$

This is again a model-level notion. It does not claim that physical time is exhausted by this quantity; it claims that, in the model, temporal extension is encoded by accumulated process cost.

Lemma 8.8. *For every morphism f,*

$$t(f) \geq 0.$$

Proof. This follows immediately from the nonnegativity of κ^{+}. $\square$

Theorem 8.9. *Every nontrivial process has positive time.*

Proof. If $t(f) = 0$, then $\kappa^{+}(f) = 0$. By the equivalence established earlier, f is a symmetry. Therefore any non-symmetry satisfies $t(f) > 0$. $\square$

Corollary 8.10. *Time appears as a derived characteristic of processual cost.*

Corollary 8.11. *The arrow of time expresses the non-symmetry of admissible processes.*

9 No-Go Theorems as Consequences of the Central Law

The standard no-go statements follow from the central law and the impossibility of effective collapse. They do not introduce additional axioms.

9.1 No-Cloning

Theorem 9.1 (No-Cloning). *Free cloning of a nontrivial object is impossible.*

Proof. Let Δ_X denote a duplication morphism for an object X. Assume that X is nontrivial and that $\kappa^{-}(\Delta_X) = 0$. Then Δ_X is a symmetry. But a symmetry preserves the observational profile and therefore does not generate a genuinely new nontrivial bearer of distinction; it only reproduces the same observational class.

If such duplication were possible at zero cost, then the model would allow distinction to be replicated without residue and without resource expenditure. This would amount to an effective collapse of the difference between original and copy into a trivial repetition of the same class, contradicting the axiom $\neg 0_{\mathrm{eff}}(E)$. Therefore free cloning is impossible. $\square$

Corollary 9.2. *Any cloning process in the nontrivial sector has positive lower cost.*

9.2 No-Erasure

Theorem 9.3 (No-Erasure). *Free erasure of a nonzero obstruction is impossible.*

Proof. This is a restatement of the information-erasure theorem, but the contradiction with the axiom can be made explicit. If a nonzero obstruction were removed at zero cost, then the removing morphism would have $\kappa^{-}(f) = 0$, hence it would be a symmetry. Symmetries preserve the obstruction class, so a nonzero obstruction cannot become zero under such a morphism.

Therefore any process that truly erases distinction must either pay positive cost or fail to erase the obstruction. A zero-cost erasure would realize the forbidden effective collapse $0_{\mathrm{eff}}(E)$. $\square$

Corollary 9.4. *Complete erasure of distinction without residue is impossible.*

9.3 No-Simulation

Theorem 9.5 (No-Simulation)**.** *A universal free simulator does not exist.*

Proof. Suppose there exists a morphism U such that for every object X,

$$\mathrm{Obs}(U(X)) \cong \mathrm{Obs}(X),$$

and $\kappa^-(U) = 0$, while U is not trivial. By $\kappa^-(U) = 0$, U is a symmetry. But a symmetry preserves the observational class and does not generate a nontrivial simulated structure beyond the original class. Hence such a universal free simulator would collapse the observational content of all objects into a trivial repetition, contradicting the axiom $\neg 0_{\mathrm{eff}}(E)$. □

9.4 No-Universal-Reduction

Theorem 9.6 (No-Universal-Reduction)**.** *There is no morphism that sends all observable structures to a single trivial class without residue.*

Proof. Such a morphism would implement $0_{\mathrm{eff}}(E)$ by definition. This contradicts the axiom $\neg 0_{\mathrm{eff}}(E)$. □

Corollary 9.7. *Every reduction is local and partial.*

10 Scale Layer and Representation Regimes

The core theory and the central law are independent of any particular scale. However, concrete scientific descriptions operate at specific scales. The scale layer is therefore not an additional ontology, but a model-level organization of how the same structural constraint appears under different coarse-grainings.

10.1 Scale Transformations

Definition 10.1 (Scale transformation)**.** *A scale transformation is a functor*

$$S : E \to E_S$$

that maps the core category into a reduced or coarse-grained category E_S.

The functor S is not assumed to be faithful or full. In general, it may identify distinct objects and morphisms.

Definition 10.2 (Scale invariance of the core)**.** *A statement is scale-invariant if it is preserved under all admissible scale transformations.*

Theorem 10.3. *The axiom $\neg 0_{\mathrm{eff}}(E)$ is scale-invariant.*

Proof. Assume that there exists a scale transformation S such that in E_S one has

$$0_{\mathrm{eff}}(E_S).$$

Then there exists a morphism in E_S that implements effective collapse. By definition of S, this morphism corresponds to a class of morphisms in E. Since scale transformations do not create new structure ex nihilo but only identify or forget distinctions, the existence of effective collapse in E_S implies the existence of an effective collapse in E. This contradicts the axiom. Therefore the axiom is preserved under all scale transformations. □

Corollary 10.4. *The central law is scale-invariant.*

Proof. The central law is a direct consequence of the axiom and the definitions of cost and obstruction. Since the axiom is scale-invariant, the law is preserved. ☐

10.2 Coarse-Graining and Residue

Definition 10.5 (Coarse-graining)**.** *A coarse-graining is a scale transformation S that identifies objects or morphisms with distinct observational profiles.*

Theorem 10.6. *Coarse-graining cannot eliminate all obstruction without introducing effective collapse.*

Proof. Suppose that a coarse-graining S maps all obstruction classes to zero. Then in E_S all compatible local families become globally liftable. This means that all distinctions are eliminated without residue, which is precisely effective collapse in E_S. By the previous theorem, this is impossible if the axiom holds in E. ☐

11 Layered Scientific Interpretation

The theory does not posit separate domains such as physics, computation, or information as independent ontologies. Instead, these domains appear as different representations of the same structural constraint.

11.1 General Principle

Theorem 11.1. *All scientific representations correspond to model-level realizations of the same underlying obstruction-cost structure.*

Proof. Each scientific domain introduces its own observables, dynamics, and invariants. In the present framework, these correspond to choices of:

1. observational family A,
2. scalarization family M,
3. scale transformation S.

Different domains differ in these choices but operate on the same underlying category E and are constrained by the same axiom. Therefore they are representations of the same structural constraint. ☐

11.2 Physical Interpretation

- Obstruction $\delta(s)$ corresponds to conserved or nontrivially structured quantities.
- Cost κ^+ corresponds to energy-like or action-like quantities.
- The central law expresses the impossibility of eliminating structure without energetic or dynamical expenditure.

11.3 Informational Interpretation

- Obstruction corresponds to information content.
- Cost corresponds to computational or thermodynamic resource.
- The central law reproduces the impossibility of free erasure.

11.4 Computational Interpretation

- Morphisms correspond to programs or computational steps.
- Cost corresponds to computational complexity.
- The central law implies that nontrivial computation requires nonzero resources.

11.5 Geometric Interpretation

- Objects correspond to points or states.
- Morphisms correspond to paths.
- Cost induces a metric structure.
- Time arises as accumulated path cost.

Remark 11.2. *These interpretations are not additional axioms. They are model-level identifications of the same formal structure.*

12 Falsifiability and Constraints

The theory is not merely structural; it makes a precise falsifiable claim.

Definition 12.1 (Empirical realization of effective collapse)**.** *An empirical realization of effective collapse is a physically or computationally realizable process that:*

1. *eliminates all observable distinction,*
2. *leaves no residual obstruction,*
3. *incurs zero cost.*

Theorem 12.2. *The existence of an empirical realization of effective collapse falsifies the theory.*

Proof. Such a realization would instantiate $0_{\mathrm{eff}}(E)$, contradicting the axiom. □

Corollary 12.3. *Any verified instance of zero-cost information erasure, zero-cost nontrivial computation, or zero-cost structural reduction would refute the theory.*

12.1 Model Constraints

Even without full falsification, the theory imposes constraints:

- lower bounds on resource consumption,
- irreducibility of certain structures,
- limits on compression and simulation.

13 Conclusion

The theory is based on a single principle:

$$\neg 0_{\text{eff}}(E).$$

From this principle, the following are derived:

1. observational structure of objects and processes,
2. resource category and cost,
3. obstruction to gluing,
4. central law of irreducible residue,
5. no-go theorems,
6. geometric and temporal interpretations,
7. unified representation of scientific domains.

No additional ontological primitives are introduced.

Theorem 13.1 (Final statement)**.** *All admissible structures preserve nontrivial distinction at a positive minimal cost.*

Proof. This is a restatement of the central law combined with the axiom. □

Remark 13.2. *The theory does not claim that all physical or computational systems are fully described by this model. It claims that any admissible system that respects distinction and transformation must satisfy the impossibility of effective collapse.*

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